Journal of Applied Mechanics and Technical Physics, Vol. 38, No. 2, 1997

INTERNAL WAVES GENERATED BY CIRCULAR TRANSLATIONAL MOTION OF A CYLINDER IN A LINEARLY STRATIFIED FLUID

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UDC 532.59

The wave pattern of the "St. Andrew's cross" type which appears upon oscillations of a circular cylinder in a stratified fluid with a constant density gradient is well known [1–6]. A characteristic feature of this pattern is that the waves are generated only in a narrow vicinity of the four rays issuing at angles $|\theta| = \arcsin(\omega/N)$ [4] to the horizontal into the four quadrants of the Cartesian coordinate system whose center coincides with the center of the cylinder in its average-in-time position. Here ω is the frequency of body oscillations, $N = \sqrt{-g\partial\rho/(\rho\partial z)}$ is the Brunt–Väisälä frequency or the buoyancy frequency, g is the acceleration of gravity, and $\rho(z)$ is the density distribution over the depth (the z axis is directed upward). The available papers report on both theoretical [1–3, 6] and experimental [1, 5–8] studies in which the source of disturbances accomplished vertical and horizontal oscillations [3, 5, 6], pulsating motion with a variation in volume [6], translational motion at an angle [7], and also combined (translational and oscillatory) motion [8].

The oscillations of a source in a stratified fluid are of interest when the trajectory of the cylinder center is a circle. This trajectory is of significance as applied to the problem of the rolling of body oscillations under the action of waves [9].

The purpose of this work is to realize and study the system of waves arising in a linearly stratified fluid upon circular translational motion of a cylinder in which the radius of the trajectory is small compared with the cross-sectional dimension of the body. It was found that under the clockwise motion of the cylinder, waves are generated only inside the band passing through quadrants I and III. It should be noted that in a uniform fluid with such a motion of the source, waves propagate near the free surface only in one direction, to the right (with clockwise rotation) [10].

Tests were performed in a test tank 100 cm long, 14 cm wide, and 30 cm high. To produce a linear stratification, 11 fluid layers with a density shift $\delta\rho = 0.002$ g/cm^3 and a thickness $\Delta h = 2.8$ cm were poured through Porolon floats floating on the surface. The total depth of the liquid was $H = 28$ cm (the upper and bottom layers were 1.4 cm thick), and the total density difference was $\Delta\rho = 0.02$ g/cm^3. After a time (~24 h), a linear density distribution was established over the vertical because of the diffusion of the admixture (glycerin) used to produce stratification [11].

Waves were generated by circular translational motion of a cylinder of diameter $D = 3.6$ cm and length 14 cm using a special mechanism. The trajectory radius was $a = 0.6$ cm, and the oscillation period T was varied. The cylinder was placed at a distance of 14 cm from the bottom and at 40 cm from the left butt wall of the test tank.

The waves were recorded by photography of the liquid body against a dull luminous screen placed behind the test tank. To visualize the wave pattern, every other layer was dyed during pouring. Photography and measurements were started after several periods of oscillations from the beginning of motion, but before the effect of reflected waves began to manifest itself. Using resistive probes with horizontal electrodes, we recorded density fluctuations at fixed points inside the liquid volume and measured the vertical density distribution in the undisturbed fluid. The principle of operation of the probes is based on the fact that an aqueous solution of glycerin is a conductor whose conductivity is directly proportional to the glycerin concentration. This makes it possible to reconstruct the density profile from the voltage decrease measured by the probe at different

Lavrent'ev Institute of Hydrodynamics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 38, No. 2, pp. 64–67, March–April, 1997. Original article submitted September 13, 1995.

 0021-8944/97/3802-0224 $18.00

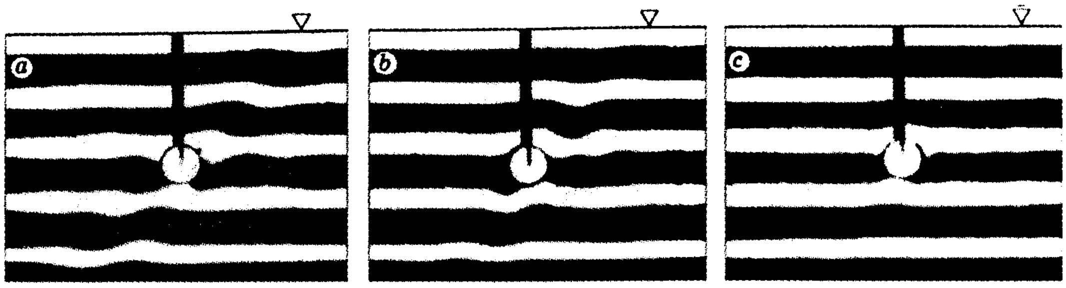

Fig. 1

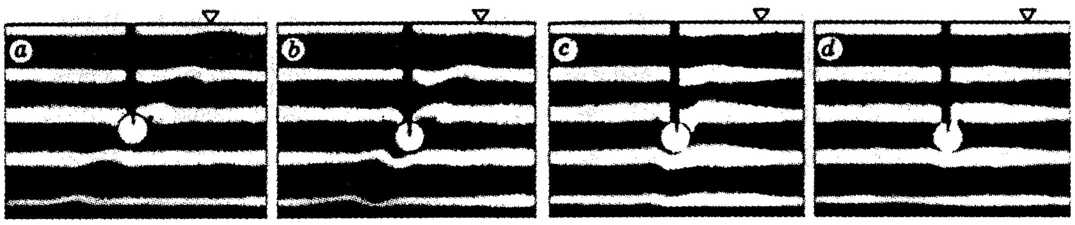

Fig. 2

points along the vertical if the densities ρ_1 and ρ_2 near the free surface and near the bottom are known. The densities were measured by standard areometers.

We performed several series of experiments. Figure 1 shows photographs obtained immediately after filling of the test tank for different oscillation periods (a–c correspond to periods $T = 9.6$, 7.8, and 6.3 sec). The $N(z)$ distribution in this series of experiments was characterized by the presence of local maxima in the region of interlayer boundaries. These maxima correspond to the buoyancy period $T^* = 4.5$ sec. The linear stratification that appears in the test tank with time has the buoyancy period $T_0 = 7.36$ sec.

In the case of layered stratification, the circular motion of the cylinder with $T > T_0$ generates waves mainly in quadrants I and III. The intensity of disturbances in quadrants II and IV is markedly weaker (Fig. 1a and b). When $T^* < T < T_0$, disturbances with a rather small wavelength propagate along the interlayer boundaries intersected by the moving cylinder. In this case, the intensities of disturbances from the right and from the left of the cylinder are practically the same (Fig. 1c). The free surface is shown by a triangle (Figs. 1–3).

Figures 2 and 3 give photographs obtained 16 and 24 hours after the filling of the test tank. In the first case, the density distribution deviates slightly from the linear distribution at the horizons that correspond to the interlayer boundary. The previous experiments in [11] showed that 19 hours after the filling of the test tank, the thickness of the zone with a linear density distribution between the layers was 2.8 cm. In these experiments, the thickness of each layer was the same. Therefore, as one would expect, after 24 hours, the density distribution becomes linear.

The photographs in Fig. 2a–c were obtained for $T = 10.8$, 9.6, and 7.8 sec; $\theta_{\rm th} = 43.3$, 50.4, and 71.6°; and $\theta_{\rm ex} = 49$, 56, and 74°; the photographs in Fig. 2d were taken for $T = 6.3$ sec; and the photographs in Fig. 3a–d were taken for $T = 10.8$, 9.6, 8.5, and 7.4 sec; $\theta_{\rm th} = 43.3$, 50.4, 60.5, and 84.5°; and $\theta_{\rm ex} = 45$, 53, 60, and 84°. The subscripts th and ex denote theoretical and experimental values for the slopes of bands that limit the wave-propagation regions. In Fig. 2, the slope of the bands exceed the theoretical values, and this is likely to be due to the presence of weak disturbances of the linear density profile. Figure 2d corresponds to the motion of a cylinder with $T < T_0$. In this case, as in the case of oscillatory motion [5–6], waves do not propagate, and disturbances are localized in the vicinity of the cylinder.

The experimentally measured slopes of the bands (Fig. 3) are very close to the theoretical values. There are no wave disturbances in quadrants II and IV for all frequencies of cylinder oscillations. This pattern might

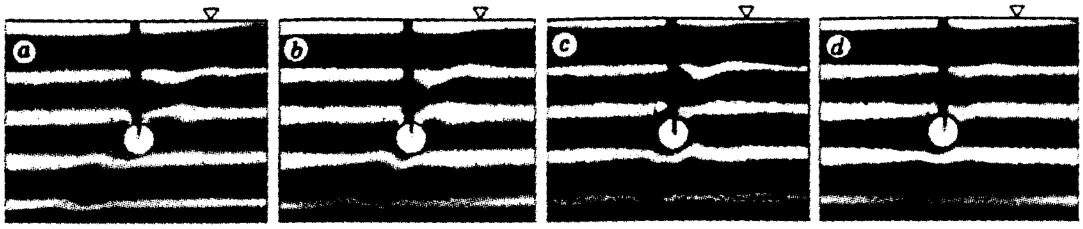

Fig. 3

be expected on the basis of the symmetry and antisymmetry properties of the fields of wave disturbances generated upon horizontal and vertical oscillations of the cylinder [3]. These properties are common for bimodal and unimodal envelopes of the particle displacement across the wave beam [6]. It should be noted that the disturbances in quadrants II and IV are also small when the cylinder reciprocates in a plane that coincides with the direction of the rays passing through these quadrants at angles $|\theta| = \arcsin(\omega/N)$. In this case, to each frequency ω corresponds a single slope of the plane of oscillations, at which waves are emitted only into quadrants I and III [6].

We performed control measurements of density fluctuations using probes placed at fixed points at a distance $R = 7$ cm from the center of the circular trajectory described by the cylinder. The conditions of the measurements correspond to Fig. 3b. It was established that the ratio of the maximum density fluctuations at fixed points in quadrants IV and I did not exceed 3.5% and was within the measurement accuracy.

In these experiments, the envelope geometry was bimodal [6] (Figs. 2 and 3), since the viscous wave scale $l = (\nu g/N)^{1/2}$ (ν is the kinematic viscosity of the liquid) is smaller than the diameter of a cylinder executing circular translational motion (in these tests, $l = 2.8$ cm).

Figures 2b and 3b correspond to the maximum intensity of wave disturbances for a constant radius of trajectory of the cylinder center. In this case, $T_0/T = 0.77$, which is in good agreement with the result obtained for vertical oscillations of the cylinder [5].

During the circular motion of the cylinder, periodic formation of high-gradient density fronts was observed. The fronts are formed when, from above and from below, the cylinder was in the flow of thin liquid layers whose density was different from the density on this horizon at rest. The motion of the liquid layer over the cylinder surface was accompanied by counterclockwise spiral swirling of its leading boundary (Fig. 2a and b). In motion with $T < T_0$, spiral structures were not formed.

The pattern of wave generation under circular motion of a cylinder suggests that the hydrodynamic loads acting on the cylinder are also "polarized" in one direction. Apparently, similar effects occur when the cylinder is stationary and the fluid motion in circular trajectories is induced by an incident internal wave. This is likely to be responsible for the "polarization" of the dynamic action of internal waves which was found experimentally in [12].

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